

## **Probabilities over What?**

### **Commentary on Tourmen**

Mark H. Bickhard

Lehigh University, Bethlehem, Pa., USA

---

#### **Key Words**

Bayesian nets · Emergent constructivism

Tourmen [this issue] gives an overview of a number of convergences and divergences between some contemporary probabilistic models of learning and development and Piaget's model. Tourmen's points are interesting and important, but so are also some caveats concerning the relationships between probabilistic models and Piaget's model, which are given limited attention. I elaborate on some of those caveats.

Models of learning focusing on the learning of probabilistic relationships have expanded over recent years. It is clear, and clear in Piaget's work, that this is an important field of thought and development. There are at least two prominent species of such models in the current literature: one derived from Bayesian models of causality and causal inference, and another known roughly as "predictive brain" models. This article focuses exclusively on the first and so will also my comments.<sup>1</sup>

Tourmen lays out very nicely multiple ways in which the Bayesian net causal model framework has some convergences with Piaget's model; I will not rehearse those here. She also mentions in one or two sentences some differences which I argue are fundamental, and, therefore, at least partially undercut some of the more expansive claims made for the Bayesian net modeling approach.

I first note that Bayes is a decision rule: it yields probabilistic information concerning the "best" selection to be made – the best decision among alternatives – in a specific Bayes' rule sense. It is a powerful decision rule, but just one among many [e.g., Berger, 2010; Ferguson, 1967]. Decision rules involve varying kinds and degrees of power, with some, for example, being (under some circumstances) specific versions

<sup>1</sup> For discussions of predictive brain models, see Bickhard [2015, in press].

of others, and they also involve varying kinds of assumptions concerning available information of relevance to the decision. Bayes' rule, for example, does not take into account the consequences (costs and benefits) of various decisions, though it can be incorporated into the formalism (but in general is not so incorporated, at least not well, in either of the probabilistic approaches mentioned above). As Tourmen [this issue] writes, "How do priors relate to goals, groups of actions, and to each other?"

A crucial assumption in Bayesian models is that the space of relevant hypotheses (or other choice alternatives) is available. This is simply an aspect of the fact that a probability distribution is distributed over something already given. Bayesian procedures, then, modify prior probability distributions over such spaces into posterior distributions over such spaces, based on current (relevant) data. They do not modify, nor generate, the spaces over which the probability distributions are distributed. Modifying the probability distribution over a space of hypotheses is not equivalent to Piagetian equilibration, for example producing new hypotheses, or new representations out of which new hypotheses could be constructed. In that sense, Bayesian models are models of confirmation and disconfirmation, *not* of the learning of new cognition or representation, or new hypotheses [as noted for learning theories in general by, for example, Fodor, 1975]. This is in contrast to Piaget, who attempted a non-foundationalist model of representation – one that was neither empiricist nor nativist. Bayes presupposes representation; Piaget did not.

Piaget attempted to model such representational emergence via a model of cognition and representation as constituted in action and potential action. Tourmen acknowledges this framework underlying Piaget's work and the fact that it marks a difference from the Bayesian framework. But I would suggest that the importance of Piaget's action framework is deeper than that: it underlies his attempt to transcend empiricism and rationalism (nativism) altogether, and this constitutes a fundamental difference from confirmatory models, such as Bayesian approaches yield.

One consequence of this difference for Piaget is that his model of the development of causal knowledge is intrinsically intertwined with his model of the development of object cognition (among others): causality is modeled relationally, and the relata tend to be either objects or events. In either case, the representations of the relata have to be themselves somehow constructed.<sup>2</sup>

I submit then that one of the most fundamental differences between Piaget's model and contemporary probabilistic models of learning and development is that Piaget attempted a "third way" beyond both empiricism and rationalism in his model of the emergence of representation, and probabilistic models, in contrast, take representation (and spaces of hypotheses constructed out of such representations) for granted, with no account of them being offered.

There are a few additional points in Tourmen's article that I would like to suggest adjusting, of which I will mention two. The first is the suggested assimilation of "necessity" to "probability 1," especially inductively estimated probability 1. This is

---

<sup>2</sup> This points to yet another difference: Piaget's model is a form of constructivism, and so also in some sense are Bayesian models, but the "construction" involved is not the same. In fact, it is rather unclear what the processes of construction are in Bayesian approaches (are modifications of probability distributions "constructions"?), but it is clear that they are not "emergent" constructivisms [Allen & Bickhard, 2011], and Piaget's is.

vastly weaker than notions of necessity as “without even possible exceptions” (e.g., try  $2 + 2 = 4$ ). It loses the modal notion of necessity entirely, and, in so doing, reveals yet again that the Bayesian models cannot stand on their own. This is at times a contentious issue, especially from various kinds of strict empiricists, but nothing offered to date accounts for this modality of necessity.

A second point concerns what kinds of phenomena are taken to contradict Piaget by Tourmen, for example:

“Even animals, like rats [Blaisdell, Sawa, Leising, & Waldmann, 2006], are able to build causal models of their environment from the observation of events and their frequency.” As is all too often the case, Piaget is discussing *mastery* of causality and probability, while these supposed disconfirmations of Piaget are based on much more primitive abilities (e.g., frequencies, associative strengths, etc.) that Piaget, to my knowledge, never denied.

And for one more example:

“First, are children attentive to causal links and frequencies earlier in development than Piaget claims?” Piaget’s claims about mastery have little to do with what Piaget would accept about children’s “attentiveness.” In fact, if children did not attend to such relevances and precursory phenomena, they could never develop any kind of Piagetian mastery.<sup>3</sup>

In summary, I applaud Tourmen’s analysis of contemporary probabilistic approaches and their similarities and differences from Piaget’s models. We need more such comparative analyses, across multiple kinds of frameworks. I also, however, wish to suggest that Piaget’s attempt at a third way emergence model of cognition and representation constitutes an underappreciated difference from probabilistic models that is of fundamental importance.

## References

- Allen, J.W.P., & Bickhard, M.H. (2011). Emergent constructivism. *Child Development Perspectives*, 5, 164–165. doi:10.1111/j.1750-8606.2011.00178.x
- Allen, J.W.P., & Bickhard, M.H. (2013). Stepping off the pendulum: Why only an action-based approach can transcend the nativist-empiricist debate. *Cognitive Development*, 28, 96–133. doi:10.1016/j.cogdev.2013.01.002
- Berger, J.O. (2010). *Statistical decision theory and Bayesian analysis*. New York, NY: Springer.
- Bickhard, M.H. (2015). Toward a model of functional brain processes II: Central nervous system functional macro-architecture. *Axiomathes*, 25, 377–407. doi:10.1007/s10516-015-9276-9
- Bickhard, M.H. (in press). The anticipatory brain: Two approaches. In V.C. Müller (Ed.), *Fundamental issues of artificial intelligence*. Berlin, Germany: Springer (Synthese Library).
- Blaisdell, A.P., Sawa, K., Leising K.J., & Waldmann, M.R. (2006). Causal reasoning in rats. *Science*, 311, 1020–1022. doi:10.1126/science.1121872
- Ferguson, T.S. (1967). *Mathematical statistics: A decision theoretic approach*. New York, NY: Academic Press.
- Fodor, J.A. (1975). *The language of thought*. New York, NY: Crowell.

<sup>3</sup> There are myriad instances of claims of refutations of Piaget based on false understandings of what Piaget wrote, and, more recently especially, a growing number of discussions pointing this out. For one such recent discussion, see Allen and Bickhard [2013].